A logical perspective on (finite) software systems and their composition

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2012-12-15
Systems - Informally

- Systems
- Informally

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A logical perspective...

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A **finite system** is defined by a tuple \( S = (T, \text{succ}, Q, I, O, x, \text{in}, \text{out}, f) \).

- \( T \) is the enumerable set of time values starting with 0 such that \( \text{succ} : T \rightarrow T \) is the invertible time successor function.
- \( Q, I \) and \( O \) are the finite sets of state values for the internal, input and output states \((x, \text{in}, \text{out}) : T \rightarrow (Q, I, O^\epsilon)\).
- \( f = (f^\text{ext}, f^\text{int}) : I \times Q \rightarrow O^\epsilon \times Q \) is a function describing the time evolution or system operation triggered by an update of its input parameters and updating the internal and output state in one time step for each \( t \in T \):
  \[
  \begin{pmatrix}
  \text{out}(t+1) \\
  x(t+1)
  \end{pmatrix} =
  \begin{pmatrix}
  f^\text{ext}(\text{in}(t), x(t)) \\
  f^\text{int}(\text{in}(t), x(t))
  \end{pmatrix}.
  \]

\( \epsilon \) symbolizes the empty character and \( I^\epsilon = I \cup \epsilon \) and \( O^\epsilon = O \cup \epsilon \).

The \( n \)-fold application of \( \text{succ} \) is written as \( t +_S n := \text{succ}^n_S(t) \).
System Composition/Super System Formation

Sequential Composition \((S_2 \circ S_1)\)

Parallel composition \((S_2 || S_1)\)
The right distribution law holds: \((P_1 || P_2) \circ S = (P_1 \circ S) || (P_2 \circ S)\)
Richer Interaction Semantics

"Interaction-Diagramm"

System 1  \(\rightarrow\)  System 2

Message \(n_1, n_2, \ldots\)

Message \(m_1, m_2, \ldots\)

System 1

System 2

System 3

System 1  \(\rightarrow\)  System 2

System 1

System 2

System 1_2

...
Proposition: Let $S$ and $U$ be two systems described by DFIOAs $\mathcal{D}$ and $\mathcal{B}$ respectively. $U$ interacts with $S$ only by the consistent protocol $\mathcal{P}(\mathcal{A}, \mathcal{B})$ with a set of final states as acceptance component, where $\mathcal{A}$ is an NFIOA describing only a projection of $S$. $S$ additionally interacts with other systems, denoted by $\sim U$, by other consistent protocols. Then $S$ and $U$ are subsystems of a larger system $\mathcal{T}$. 
Recursive System Relations

System $\mathcal{U}_1$

```c
int fac1(int i) {
    if (i==0)
        return 1;
    else
        return i*fac2(i-1);
}
```

System $\mathcal{U}_2$

```c
int fac2(int i) {
    if (i==0)
        return 1;
    else
        return i*fac1(i-1);
}
```
Summary

The effect of system interaction on system composition can be classified as:

- Parallel processing or strict sequential interaction results in strictly hierarchical super system formation
- Deterministic bidirectional interactions together with certain consistency conditions result in (recursive) super system formation
- Nondeterministic bidirectional interactions together with certain consistency conditions results in provably no super system formation
- further, non-classified relations.

Literature:
Thank You!

Any questions?

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